87. Acoustic waves and measurement of the sound velocity

Foreword

This lab course experiment is designed to be performed at home with everyday available tools. As you are building the setup on your own and finding individual solutions on how to perform the measurement, it is vital to take notes in form of a lab book. Your report should explain not only the physics behind the measurement but also the methods you chose. Furthermore, attach the raw data as well as a copy of the lab notes in the appendix of your report.

Requirements:

- smartphone with the *PhyPhox*-App installed
- loudspeaker (a single one or the loudspeaker from your laptop or computer)
- cardboard tube (e.g. where your aluminium foil for the kitchen is wrapped around)
- ruler or measure
- headset (or another small microphone)
- wall
- a quiet environment
- a thermometer (if you do not have one, you could try to estimate the temperature)

I. Introduction

In this lab course experiment, you will study waves and interference phenomena using acoustic waves. In contrast to electromagnetic waves, acoustic waves have a much longer wavelength and allow to study wave phenomena on a millimeter range rather than a micrometer range, which makes it particularly suited for experiments with everyday technology.

In this experiment, we will think about the properties of acoustic waves and compare them to electromagnetic waves. You will detect sound with a microphone and study its frequency spectrum as well as phenomena like interference and resonance. Using these techniques you will measure the sound velocity with a simple yet precise setup and study imperfections in the experimental setup. In a second part you will characterize acoustic waves in space.

II. Preparation

1. Useful references for the preparation

In order to prepare for the topics sound wave, sound detection with a microphone, sound pressure level (SPL), frequency spectrum, spectral feature of different acoustic noise sources, resonance conditions of an open tube, sound velocity in gases, acoustic impedance matching, standing waves and fast Fourier transform, the following references might be useful:

- Wikipedia entries about Sound, Acoustic wave, Speed of sound, Acoustic resonance and Fundamental frequency
- Hirth, M., Kuhn, J., Müller, A. (2015). Measurement of sound velocity made easy using harmonic resonant frequencies with. Phys. Teach., 53(120), 2013–2015. https://doi.org/10.1119/1.4905819
- Press, W. H, et al., Numerical Recipes The Art of Scientific Computing, Third Edition, Cambridge University Press (2007), Chapter 12 (Fast Fourier Transform)

2. Questions for the students

The following questions should test your knowledge and guide you through the preparation for performing the experiment:

a) Acoustic wave

- What are the differences between longitudinal and transversal waves?
- What is a sound waves? Is it a longitudinal or a transversal wave?
- How does the pressure distribution and the particle displacement evolve in time and space for a propagating wave?
- What determines the sound velocity? What external parameters influence its value?
- What is the sound velocity in air?
- How can one generate and detect sound?
- How is the sound pressure level defined?

b) Wave phenomena in acoustics

- How long is the wavelength of an acoustic wave in air with a frequency of 1000 Hz?
- Can an acoustic wave be reflected, refracted or attenuated, like other kinds of waves? If yes, in which situation?
- What do you expect to observe when an acoustic wave gets reflected from a wall and travels opposite to the incoming wave?
- What influence does the boundary have on the pressure distribution and on the particle displacement?

c) Resonance inside an open tube

- What is an acoustic resonance? In which situations do you expect to see resonances?
- Which frequency spacing does a resonator have? How is this related to the harmonic series of music instruments?
- Do you expect to see resonances for an open tube and why?
- Which relation between the fundamental frequency of an open tube and a tube closed at one side with the same length do you expect?
- Do the resonance frequencies of an open tube and a tube open on one end depend on the diameter?
- How does the acoustic wave look like for different resonances inside the tube (open and partly closed)? Consider pressure distribution and particle displacement.
- What is acoustic impedance matching?

d) Fast fourier transform of a digital signal

- How does one measure an analog signal with a digital device? Which values are discretized? What is a sampling rate?
- What is the Nyquist limit or Nyquist frequency?
- What determines the highest and lowest frequency that one can measure with a digital audio signal?

III. Tasks

- 1. Resonance frequencies in a tube
 - (a) Measure the resonance frequency spectrum of a tube that is open on both ends.
 - (b) Extract the fundamental resonance frequency by fitting a Lorentz peak to the data.
 - (c) Determine the resonance frequency of a tube that is closed on one end.
 - (d) Compare the resulting fundamental frequencies for the close and open tube.
- 2. Measurement of the sound velocity based on the resonance frequencies
 - (a) Fit the resonance frequency data sets for the open tube and the tube closed on one end with a multi-lorentian peak fit to extract also higher order resonance frequencies.
 - (b) Plot the obtained resonance frequencies as a function of their resonance order in two separate plots and use a linear fit to extract both slopes.
 - (c) Calculate the speed of sound for the two measurements.
 - (d) Compare the obtained values with the theoretically expected sound velocity for the ambient parameters, i.e. the temperature of the air.
- 3. Characterizing an acoustic standing wave
 - (a) Excite the tube on the resonance frequency with a single-frequency source and measure the acoustic standing wave inside an open tube. Do this by sliding the microphone of the headset through the tube while aquiring the sound pressure level. Perform the measurement for at least three different resonance frequencies of your tube.
 - (b) Measure the acoustic standing wave between a frequency source and a wall by sliding the microphone between the wall and the frequency source and aquiring the sound pressure level. Perform the measurement for at least two different frequencies.
 - (c) Determine the position of the minima in the sound pressure level for at least two different frequencies of the standing wave. Compare the obtained minima positions with your theoretical expectation based on the measured sound velocity.

IV. Performing the experiments

In the following, we will discuss the setups required for performing the experiment, and we will give hints for data taking in order to mitigate possible problems and potential sources of errors. The data acquisition can be done using the smartphone app *PhyPhox*. A guide to the app as well as a description of the data analysis can be found in the chapter thereafter¹.

1. Resonance frequencies in a tube

a) Resonance frequencies of the open tube

In the first part, resonance phenomena shall be investigated using a tube open at both ends, and a tube open only on one end. The execution of the experiment will be rather simple but the analysis of the data should be carried out carefully and is more complex.

To perform the experiment, we will excite the resonance frequencies of the air inside the tube. One possibility to do this is to excite with a broad acoustic spectrum. Coupling of the broad spectrum to the resonance frequencies then allows to detect the resonances at the other end of the tube. The measurement can be done by positioning the one end of the tube close to the microphone of your smartphone and record the frequency spectrum without any excitation to see the background signal. What do you observe? Then, try to see the resonance peaks showing up while you blow over the other edge of the tube.

Due to the resonator having open ends its acoustic length is longer than the tube length by about 0.6133Ron each side where R is the tube radius [3]. Make sure that the microphone is positioned by a factor 3-4 further apart from the tube end than 0.6133R when you perform the measurement in order to not alter the acoustic length with your measurement device. Once you see that the excitation technique works, record a data set



Figure 1: Experimental setup: the noise cause by crumpling paper generates standing waves in a tube. The resonant frequencies are plotted by a spectrogram using a smartphone or tablet PC. Alternatively, the noise can be caused by blowing at the edge of a tube or by a computer loudspeaker. Taken from [2]

of 500 ms length with the *audio scope* function using the same excitation method. Export the data to your pc and perform a Fourier transform of the audio track to obtain the information in frequency space. In principle recording the spectrum with the audio spectrum function is equivalent to recording the track with the audio scope and then Fourier transforming it, but it turns out that the audio spectrum function has glitches in the signal. Therefore the data will be more reliable taken with the *audio scope*.

Extract the fundamental frequency by fitting a lorentian function to the data

$$l(f) = y_0 + \frac{2A}{\pi} \frac{w}{4(f - f_0)^2 + w^2}$$
(1)

where y_0 is the offset, f_0 is the center position, w is the width of the peak and A is the area under the peak.

¹Of course, also other smartphone apps can be used. The vital features of such an app are: recording acoustic frequency spectra, recording the sound pressure level and exporting the data for further data analysis.

b) Resonance frequencies of the tube closed on one side

To perform this experiment, you can close the tube with a plastic cap or a piece of cardboard or simply with the palm of your hand. Like before, position the microphone at the open end of the tube with the above-mentioned distance and excite the resonance frequency while observing the frequency spectrum with the PhyPhox app. Once you found a good way of exciting the resonances, record the audio track of 500 ms length with the *audio scope* and Fourier transform the data on your PC. Extract the peak position of the fundamental frequency like before and compare your result with the fundamental frequency of the open tube.

Hints and challenges It is challenging to excite the resonance spectrum without adding additional noise to the measurement. One possibility to avoid noise is to excite by rubbing the palm of the hand along the closed end of the tube.

2. Measuring the speed of sound

One can extract the speed of sound in air from the resonance spectrum of a resonator filled with air. For an open tube, the fundamental frequency f_0 as well as the higher orders f_k are given by

$$f_0 = \frac{c_s}{2(L+2a)} \tag{2}$$

$$f_k = (k+1)f_0$$
 for $k = 0, 1, 2, \dots$ (3)

where c_s is the speed of sound in air, L is the length of the tube and a = 0.6133 R is the correction to the acoustic length of the tube due to its finite diameter 2R, as we already mentioned above [3].

In contrast to that, resonance frequencies f_k of the tube open only on one end are given by

$$f_0 = \frac{c_s}{4(L+a)} \tag{4}$$

$$f_k = (2k+1)f_0 \text{ for } k = 0, 1, 2, \dots$$
 (5)

Due to the boundary condition given by the closed end, the fundamental resonance wavelength is longer by a factor of two. Additionally, the correction of the length is only a.

In principle, knowing the fundamental resonance frequency would be sufficient to determine the speed of sound. However, the results gets more precise, if we also use the higher order resonances. To determine the speed of sound based on the resonances, fit the frequency spectra with a model function that contains an addition of several lorentian peaks including an offset and an exponential decay $b_0 \exp(f/b)$ that empirically accommodates for the detection noise, where b is the decay constant and b_0 is the amplitude

$$m(f) = y_0 + b_0 \exp\left(\frac{f}{b}\right) + \frac{2A_0}{\pi} \frac{w_0}{4(f - f_0)^2 + w_0^2} + \frac{2A_1}{\pi} \frac{w_1}{4(f - f_1)^2 + w_1^2} + \dots$$
(6)

As the fit has many free parameters, calculate the theoretically expected resonance frequencies and use them as a starting condition for the fit.

The fitted peak positions can now be plotted as a function of the resonance order. To determine the sound velocity, fit a linear function without offset through the data points and extract the slope. To do this, you have to map the order of the resonance to a new index. For the open tube $(k + 1) \rightarrow k'$ where k' = 1, 2, 3... and for the closed tube $(2k + 1) \rightarrow k'$ where k' = 1, 3, 5...

Compare your result to the literature value and discuss possible error sources.

3. Characterizing an acoustic standing wave

In the last part, we want to visualize the sound waves for the resonance condition inside the tube and in case of a standing wave between a frequency source and a wall [1].

a) Acoustic waves inside the tube

Measure the acoustic wave inside the tube by mapping out the sound pressure level as a function of the position in the tube. For this, place your headset inside the tube such that you can slide it from one end to the other. Then use a tone generator app from the web and generate a single frequency at the second harmonic resonance frequency of the tube. Use the *audio amplitude* function of PhyPhox to record the sound pressure level as a function of time and slide the microphone from one end of the tube to the other with constant velocity while recording.

Repeat the same experiment with the fundamental resonance frequency and a higher harmonic of your choice. Plot all three measurements in the same graph. To qualitatively compare the features, scale the time axis such that the points of time for the microphone reaching the left or right end of the tube fall on top of each other for each measurement, respectively.

b) Acoustic standing wave at a wall

For the measurement of the acoustic standing wave, you need a loudspeaker that provides the single-frequency source, a wall that acts as an acoustic reflector and your smartphone as a detector. If you have a headset microphone, you can use it as your detector instead of the microphone built into the smartphone. Position the source at a distance of approximately 60 cm to the wall and let it emit a single frequency of your choice. Before you start the experiment, estimate the wavelength of the acoustic wave at the chosen frequency. It is convenient to start with a wavelength of about 10 to 15 cm. To measure the distance to the wall, place a ruler or measure between the wall and the source.

Measure the sound pressure level as a function of the distance to the wall. To do this, acquire the sound pressure level as a function of time with the *audio amplitude* function and move the microphone between the wall and the source with a constant velocity such that you can later map the time to the microphone position.

Hints and challenges This measurement is very challenging. It is important that there is no other reflecting surface in close proximity or at a distance on the scale of the measurement. One way of achieving this is to position the loudspeaker on a table 60 cm away from a wall and much further away from any other wall. The measurement works best with the headset microphone installed on a stick such that the disturbance of the sound wave is minimal. By moving the microphone with a constant velocity away from the wall and recording the audio amplitude as a function of time with the PhyPhox app, you can see the



Figure 2: Measurement of acoustic standing wave

minima and maxima of the acoustic wave. You will probably have to try out different trajectories of the microphone to get good data.

Perform the measurement for at least 2 different frequencies, and plot them in one plot. A quantitative analysis of the data is difficult, since the position of the microphone is usually not very precisely known, and will be performed in the next task. However, discuss in your report the shape of the SPL and what the SPL at the wall is, and why.

c) Quantitative measurement of the standing wave

As the previous measurement was challenging and at the same time quite qualitative, perform a more quantitative measurement of the standing wave by determining the position of the minima in the sound pressure level between the wall and the frequency source. To do this, use again the audio amplitude function and slide the microphone away from the wall until you detect a local minimum in the SPL. Find the position of that minimum by shifting the microphone back and forth to find the exact position where the SPL has the lowest value. Perform this measurement for at least two different frequencies and find all the minima. Sort the minima in rising order of distance to the wall and number them. Then plot the position of the minima as a function of their number. Discuss what you observe?

At what distances from the wall do you expect the nodes of a standing wave depending on its frequency? Where do you expect the minima of the sound pressure level of the standing wave?

Plot the theoretically expected positions of the minima in the SPL with respect to the wall together with your measured results for the two frequencies. Use your experimentally determined sound velocity for the theoretical curve. Is the measurement in agreement with your expectation?

V. Data recording and analysis

In the following, we give a short introduction on acquiring data with the *Phyphox* software. In the second part, we will give examples on how to import the data into *Python* and display examples for code in *Python* to analyse the data.

1. Data taking with PhyPhox

To take the data, you can use the smartphone app *PhyPhox* (https://phyphox.org/). The app allows to read out different sensors and record signals from the microphone. It further allows to store the data and export it on your personal computer in different data formats. To not disturb the experiment you can access the app remotely via a web browser to start and stop measurements. The web browser also allows you to export the data directly and access further information. On top of that, the platform allows to program small experiments directly on the phone or more elaborate ones using the pc, but we will not use that functionality.

a) Measuring with PhyPhox

To do a measurement you can choose a function and then record the data by pressing the "play" button (see Figure 3). To delete date, you can choose the symbol "dustbin". Further settings and information can be accessed with the button on the top right (three dots). In "Experiment info" you find a short description of the function and often a link to the PhyPhox wiki page where also problems and resolutions are described.

A measurement function often has different ways to show the data which you can chose with the available tabs. For specific functions, you can also do a calibration, change the setting of the data taking or read on FAQ by choosing the corresponding tab.

b) Audio functions in PhyPhox

For the measurements, we will use the microphone as our sensor. PhyPhox provides several useful functions to extract the data. We will use the functions Audio Amplitude, Audio Scope and Audio Spectrum.

Audio Amplitude extracts the amplitude of sounds by measuring the sound pressure level (SLP) in dB as a function of time with a rather low temporal resolution. We will use this

function to measure relative differences in the sound pressure as a function of time. This sensor is not calibrated and such a calibration is very elaborate. The suggested calibration scheme is not very precise as it does not take into account the dependence of the exact spectrum. As we are anyways only interested in relative differences, we do not calibrate the sensor for your measurements.

Audio Scope directly records audio signal from the microphone with a high temporal resolution and for a maximum duration of 500 ms per track.

Audio Spectrum records short tracks of audio from the microphone and calculates the Fourier Transform in real time. The length of the track is limited to a number of samples that one can set.

c) Remote access

To activate the remote access, choose a function in Phyphox, e.g. the audio amplitude and click on the button on the top right. When you choose the option "enable remote control" and click "ok" for the savety warning, you find a note on the bottom of the screen that states the URL of your device. You can enter this static address in the browser of your pc and thereby access the phone from the pc.



Figure 3: The function *Audio Scope* on Phy-Phox.

d) Data export

You have several possibilities to export a recorded data set:

- 1. direct export: You can export the data via "Export Data" via the button on the top right and choose your favored data format. After that, depending on the apps installed on your phone, you can choose different options for sending the data, like e.g. sending via email or storing on Google Drive or Dropbox.
- 2. export data set: You can also click on the graph that shows the recorded data. In a window that pops up at the bottom of the screen, you can click on "More tools" and then choose "Export this dataset". Similar to before, choose a data format and then the way how you want to transfer the data. This method is extremely useful, if you record a sonograph.
- 3. remote export: If the remote access is activated before or directly after data taking, you can export the data via "Export Data" via the button on the top right and choose your favored data format. A window will open to open the data or save it in a specified folder.

2. Data import into Python

For your python program, the following classes are useful:

```
import numpy as np
import xlrd
import matplotlib.pyplot as plt
```

```
4 import os
5 import xarray as xr
6 from lmfit import Model
```

The class os allows to find out in which folder the python code is running. The command os.getcwd() prints the folder address.

To export the data from PhyPhox, you can choose different data formats. The easiest is to store the data into the same folder that contains the python script. Then you do not have to give the full path name for the data import. To import data from an excel table into Python, you can use the class xlrd and following lines

```
1 # Use xlrd for reading excel files
2 # Give the location of the file
3 workbook = xlrd.open_workbook(r'Messung06-2020-03-24_10-04-14.xls')
4 sheet06 = workbook.sheet_by_index(0)
5 #print(sheet.cell_value(1, 0))
6
7 # Extract columns from excel format
8 time06=sheet06.col_values(0)[1::]
9 ampl06=sheet06.col_values(1)[1::]
```

3. Data plotting into Python

For plotting the data, we suggest to use the class matplotlib.pyplot. Here is an example for a configured and labeled plot

```
1 plt.figure(figsize=(7.5,5))
2 plt.plot(time06,amp106,label='test-06')
3 #plt.plot(time07,amp107,label='test-07')
4 #plt.xscale('log')
5 #plt.yscale('log')
6 #plt.axvline(x=10,linewidth=1, color='r',ls='dashdot')
7 plt.legend(loc=3)
8 plt.xlim(0,400)
9 plt.ylim(-0.2,0.2)
10 plt.xlabel('Time (ms)')
11 plt.ylabel('Amplitude');
12 plt.title('Test 06')
13 plt.show()
```

4. Data analysis with Python

The signal from the audio scope can be fourier transformed using the function fft in numpy:

```
#Fourier transform of audio track
freq_fft=np.fft.fftfreq(len(time06),(time06[2]-time06[1])/1000)
mpl_fft=np.abs(np.fft.fft(ampl06))
d
d
d
d
d
d
d
freq_fft_plot=int(len(time06)*0.5)
freq_fft_plot=freq_fft[:data_length]
mpl_fft_plot=ampl_fft[:data_length]
```

The third line is the actual fast fourier transformation (FFT) of the digital audio track signal. The second line is used to define the correct frequency column based on the initial audio track. It simply takes the length of the data set as the first parameter and the time interval Δt in seconds between two data points as the second parameter and calculates the frequency axis that fits with the FFT from line 3. The result after this step is data with negative and positive frequencies and a maximum frequency of $1/(2\Delta t)$ corresponding to the Nyquist frequency. The negative frequency data are attached behind the positive frequencies and are redundant in our case. Therefore we cut them from the data set in line 6 and 7.

For fitting the data, you can for example use the class <code>lmfit</code> where you can define a fit model with input parameters and do a least-square fit to the data

```
1 # Define the fit model
2 def Lorentz(x,y0,a,xc,w):
      return y0+(2*a/np.pi)*(w/(4*(x-xc)**2 + w**2))
3
4
5 lmodel = Model(Lorentz)
6
7 #fit the model to the data
8 #fplot is the data, tplot is the x-axis
9 #the starting parameter for the fit should be set based on the data
10 result = lmodel.fit(fplot, x=tplot, y0=0.8, a=100, xc=525, w=20)
  conf = result.eval_uncertainty(sigma=1)
11
13 # print the fit results
14 print(result.fit_report())
15
16 # Extract fit result for further usage
17 f0=result.best_values.get('xc')
18 f0err=np.sqrt(result.covar[2,2])
```

By using a model with multiple lorentians and an exponential decay, you can perform the elaborate fit of the resonance frequencies. Also a linear function can be defined as a model.

VI. German expressions for this experiment

Kartonröhre	cardboard tube
Schalldruckpegel	sound pressure level
Schallgeschwindigkeit	sound velocity, speed of sound
Abtastrate	sampling rate
Zweite Harmonische	second harmonic
Höhere Harmonische	higher harmonics
Grundfrequenz	fundamental frequency
ganzzahlig	integer
ganzzahliges Vielfaches	integer multiple

VII. Guiding questions for the report

- Discuss the relation between the fundamental frequencies of the open tube and tube open on one end.
- Compare the results for the sound velocity of the two tubes and the two configurations.
- Discuss the qualitative measurement of the standing waves in the tube and in front of the wall considering the quantity you measure and the observation. Make a suggestion for an improved setup.
- Discuss the agreement between the measurement of the nodes in the standing wave, i.e. the minima that you measured and your theoretical expectation.

References

- [1] Sammlung der Vorlesungsexperimente ETH Zuerich. Vorlesungsexperiment 5.6.7 Stehende Welle. https://expweb.phys.ethz.ch/05/06/07/bes.pdf.
- [2] Michael Hirth, Jochen Kuhn, and Andreas Müller. Measurement of sound velocity made easy using harmonic resonant frequencies with everyday mobile technology. *The Physics Teacher*, 53(2):120–121, 2015.
- [3] Harold Levine and Julian Schwinger. On the radiation of sound from an unflanged circular pipe. *Physical review*, 73(4):383, 1948.